

# Phoenix Christian Sr High School

## MTH410H AP Calculus BC

### Course Scope & Sequence

#### **COURSE DESCRIPTION**

**Goal of the Mathematics Program:** Provide students with a well-rounded base of mathematical knowledge that they will be able to apply in a variety of contexts. Underscore the hand of God in the creation through mathematics.

**Calculus Objectives:** Study the mathematics of change through limits, differentiation and integration using analysis, graphs and numerical methods. Employ the “Rule of Four”, using analytic, numerical, graphical and verbal methods, to represent and solve problems. Use hand-held graphing technologies appropriately to enhance and deepen understanding of concepts.

**National Council of Teachers of Mathematics (NCTM) Standards:** NCTM standards are referenced after each quarter.

#### **REQUIRED TEXTS AND \*KEY SUPPLEMENTAL MATERIALS**

Finney, Ross, Franklin Demana, Bert Waits, Daniel Kennedy, *Calculus: Graphical, Numerical, Algebraic*, Glenview, Illinois: Prentice Hall, 2003.

Anderson, Frank A., *Review for the AP Calculus Examination: The Two Week Difference*, Woodbury Forest, Virginia: Andco Educational Services, 1998.

Antinone, Linda, Thomas Dick, Kevin Fitzpatrick, Michael Grasse, Mark Howell, *Explorations: Calculus Activities*, Dallas, Texas: Texas Instruments, Inc., 2004.

Foerster, Paul A., *Calculus Explorations*, Berkeley, California: Key Curriculum Press, 1998.

Kamischke, Ellen, *A Watched Cup Never Cools*, Berkeley, California: Key Curriculum Press, 1999.

Guillen, Michael, *Bridges To Infinity: The Human Side of Mathematics*, Los Angeles, California: Jeremy P. Tarcher, Inc., 1983.

AP Central Web site: [apcentral.collegeboard.com](http://apcentral.collegeboard.com)

## COURSE SCOPE AND SEQUENCE

### First Quarter

#### Prerequisites for Calculus 1 Week

*Key Concepts:* Lines, Functions and Graphs, Piecewise Functions, Parametric Equations, Changing Parametric Equations to Rectangular Form, Functions and Logarithms, Trigonometric Functions, Trigonometric Identities.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes.

#### Limits and Continuity 2 Week

*Key Concepts:* Rates of Change and Limits, Left and Right Hand Limits, Limits Involving Infinity, End Behavior, Continuity, Rates of Change and Tangent Lines, Free Fall.

*Applications Project:* Students use the language of limits to describe the behavior of a given graph at its ends and near its asymptotes, along with other descriptive clues about the behavior of the graph. Students trade descriptions with other students and try to recreate the other students' graphs from the descriptions.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

#### Derivatives 3 Weeks

*Key Concepts:* Derivative of a Function, Graphs of Derivatives, Differentiability, Rules for Differentiation, Product and Quotient Rules, Velocity and Other Rates of Change, Interpreting Velocity Graphs, Derivatives of Trigonometric Functions, Derivatives of Exponential Functions, Derivatives of Logarithmic Functions.

*Applications Project:* "Graph Match" Student groups are given four sets of cards with pictures of original functions, pictures of the derivatives of those functions, verbal descriptions of the original functions, and verbal descriptions of the derivative. The groups of students must match up the cards according to  $f(x)$ ,  $f'(x)$  and their verbal descriptions.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

#### Applications of Derivatives 3 Weeks

*Key Concepts:* Extreme Values of Functions, Mean Value Theorem, Maximum and Minimum Values, Connecting the Graph of the First Derivative with Original Function, Connecting the Graph of the Second Derivative with the Original Function, Modeling and Optimization with Perimeter and Area, Modeling and Optimization with Volume, Linearization, Newton's Method, Related Rates Involving Triangles, Related Rates Involving Spheres and Distance, Related Rates Involving Cones.

*Applications Project 1:* "Bouncing Ball" Students use a motion detector and TI-84 calculator to gather data on distance, velocity and acceleration for a bouncing ball. By studying the graphs they formulate relationships between distance, velocity and acceleration. Using regression equations they test their theories analytically.

*Applications Project 2: "TRP Volume"* Students calculate the rate of change of the radius of a Tootsie Roll Pop (TRP). They collect data on the change of the radius with respect to time as they consume the TRP and use the data to calculate their personal 'TRP Force'. They use this model to make predictions about the rate of change of the volume.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

*NCTM Standards:* Understand numbers, ways of representing numbers, relationships among numbers, and number systems; understand patterns, relations, and functions; use mathematical models to represent and understand quantitative relationships.

*Biblical Integration:* Read "A Certain Treasure" from *Bridges to Infinity*. Write 500-word essay on ideas presented including how they relate to a Biblical world-view. Discuss the meaning of logic and proof and how they point to order placed in the creation by its Creator.

## Second Quarter

### The Definite Integral 3 Weeks

*Key Concepts:* Rectangle Approximation Method, Estimating with Finite Sums, Integral as Net Change, Definite Integrals and Antiderivatives, Fundamental Theorem of the Calculus, Analyzing Graphs, Trapezoidal Rule.

*Applications Project:* Students create tables and graphs for a function defined as an integral using their list menus and scatter plots. They note the location of relative extrema by examining sign changes in their lists and make the connection that the derivative of the accumulation function is the integrand.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

### Differential Equations and Mathematical Modeling 4 Weeks

*Key Concepts:* "Undoing" the Derivative, Slope Fields, Initial Value Problems, Euler's Method, Integration by Substitution, Substitution with Trigonometry, Substitution with Initial Value Problems, Integration by Parts, Repeated Integration by Parts and Tic Tac Toe Method, Exponential Growth and Decay, Logistic Growth.

*Application Project:* "Intro to Slope Fields" Each student is given a 3x5 card containing an x,y point. Students calculate the slope for their point using a given differential equation then draw a short line with that slope on a grid on the board. When all the slopes are on the grid students find the particular solution using their point. Various solutions are superimposed on the slope field they created. They also find the approximation by Euler's method for a given value of x.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

*NCTM Standards:* Analyze change in various contexts; use visualization, spatial reasoning and geometric modeling to solve problems; apply appropriate techniques, tools and formulas to determine measurements.

*Biblical Integration:* Read "Locating the Vanishing Point" in *Bridges to Infinity*. Write a 500-word essay on the ideas presented including how they relate to a Biblical worldview. Discuss the idea of limit and how it relates to our finite natures in contrast to the infinite nature of God.

### Third Quarter

#### Applications of Definite Integrals 3 Weeks

*Key Concepts:* Integral as Net Change, Work, Areas in the Plane, Volumes Using Cross-Sections, Volumes Using Disk and Washer Methods, Volumes Using Shell Method, Length of Curves.

*Application Project:* Students use clay to make a model of a solid with a given base and cross-section. They calculate the exact volume of their solid using integration.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

#### BC Integration Techniques 2 weeks

*Key Concepts:* L'Hopital's Rule, Improper Integrals, Convergence, Direct Comparison Test, Limit Comparison Test, Partial Fractions.

*Applications Project:* Students investigate finding distance traveled using two different velocity equations having a horizontal asymptote of  $t = 0$ . Calculate distances algebraically and notice the pattern as  $t$  increases without bound. One function will approach a limit, but the other will not. Make conjectures on the findings and write out conclusions.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

#### Infinite Series 4 weeks

*Key Concepts:* Geometric Series, Power Series, Maclaurin and Taylor Series, Taylor's Theorem with Remainder, Remainder Estimation Theorem, Tests for Convergence, Radius of Convergence, Interval of Convergence.

*Application Project:* Students do a graphing calculator investigation of the ability of the Maclaurin Series to approximate the value of  $y = \sin x$  using an increasing number of terms. They do the same investigation for  $y = \tan^{-1} x$  and its Maclaurin Series, noting interval of convergence.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

*NCTM Standards:* Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships; apply transformations and use symmetry to analyze mathematical situations; select and use appropriate statistical methods to analyze data.

*Biblical Integration:* Read “An Article of Faith” in *Bridges to Infinity*. Write a 500-word essay on the ideas presented including how they relate to a Biblical worldview. Discuss the importance of worldview in formulating and interpreting hypotheses in the sciences and the relevance of Scripture to the way observations are interpreted.

#### **Fourth Quarter**

Parametric, Vector and Polar Functions 2 weeks

*Key Concepts:* First and Second Derivatives of Parametric, Vector and Polar Functions, Vector Operations, Derivatives and Motion, Lengths of Parametric and Polar Curves, Area and Surface Area of Polar Curves.

*Application Project:* Students create their own polar curve and graph it on polar graph paper. They find its length, area and surface area. They then make an artistic presentation of their curve and give it a distinctive name.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.

AP Calculus Test Prep: Intensive Review of Concepts 2 weeks

*Key Concepts:* Limits, Continuity, Increasing and Decreasing Functions, Relative Extrema, Concavity and Points of Inflection, Slope of Tangent Line, Graphical behavior of  $f$ ,  $f'$ ,  $f''$ , Definition of derivative, Mean Value Theorem, Rolle's Theorem, Linear Approximation, Related Rates, Fundamental Theorem of the Calculus, Functions Defined as Integrals, Average Value of a Function, Particle Motion, Initial Value Problems, Riemann Sum and Area, Trapezoidal Rule, Area and Volume as Integrals, Exponential and Logistic Growth Differential Equations, Slope Fields, Euler's Method, Sequences and Series, Calculus of Parametric, Polar and Vector Functions.

*Assessments:* 4 Weekly Homework Assignments, 4 Weekly Quizzes, 1 Test, 2 Applications Projects.

AP Calculus Test Prep: Pulling It All Together 2.5 Weeks

*Key Concepts:* Formula and concept memorization drills: “What You Must Know Cold”, in-class group work on past AP Calculus free response questions, grading according to the AP rubric.

*Applications Project:* Groups of students are given copies of the student samples of selected AP Calculus free response questions. They find any errors and grade the samples according to the AP Rubric.

*Assessments:* 4 weekly homework assignments: past AP Calculus free response questions, 4 weekly quizzes: past AP Calculus multiple choice questions, 1 full length past AP Calculus test, applications project.

*NCTM Standards:* Build new mathematical knowledge through problem solving; make and investigate mathematical conjectures; communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

*Biblical Integration:* Read “The Call of the Wild” in *Bridges to Infinity*. Write a 500-word essay on how the ideas presented relate to a Biblical worldview. Discuss the presence of rules in game theory and how human behavior reacts to rules. Look at Scriptural teaching about sin and human responsibility.

Topics in BC Calculus Presentation

2.5 Weeks

BC Calculus students present these topics to students in the AB Calculus course, and are responsible for planning, presenting and assessing the material.

*Key Concepts:* L'Hopital's Rule, Partial Fractions, Parametric Functions, Polar Coordinates, Polar Graphs, Calculus of Polar Curves.

*Assessments:* 4 weekly homework assignments, 4 weekly quizzes, 1 chapter test, applications project.